FSK324 Test 2
Solid State Physics

Time: 90 minutes	**Total marks: [70]**

Section A - The thermal properties of materials: *Answer **ALL** questions in this section.*

1. The energy of a system in the Debye model is

$$E = 9RT\left(\frac{T}{\theta_D}\right)^3 \int_0^{\theta_D/T} \frac{x^3}{e^x - 1} dx, \quad \text{where } x = \frac{\hbar\omega}{k_B T}. \tag{1}$$

   Determine the specific heat C_v at

   i.) low temperatures [3]

   ii.) high temperatures [3]

2. With regards to specific heat capacity C_v, discuss the failings of

   i.) the classical Petit-Dulong model [3]

   ii.) the Einstein model [3]

3. The wave equation for elastic waves in a rod is given by

$$\frac{\partial^2 u}{\partial x^2} = \frac{\rho}{Y}\frac{\partial^2 u}{\partial t^2}. \tag{2}$$

   i.) Define *all* the symbols and their S.I. units in equation (2). [3]

   ii.) assuming a solution of the form $u = u(x,t) = Ae^{i(kx-\omega t)}$, show that the dispersion relation is $\omega = v_s k$ where $v_s = \sqrt{Y/\rho}$. [3]

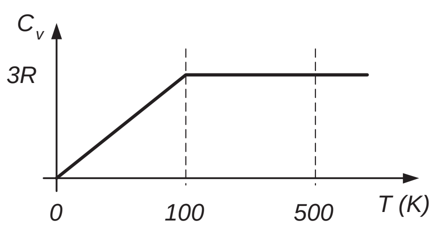

Figure 1: C_v as a function of temperature for some unknown material.

4. Using the data in Fig. 1, calculate the internal energy of 1 mole of the material at 500K. [6]

5. Given that the atomic mass and density of gold are 197 and 19.3 g/cm^3 respectively, determine the Debye temperature of gold if sound travels through it at 2.1 km/s. [6]

Section B - The free–electron model
Answer ***ALL*** *questions in this section.*

6. Describe the Hall effect in metals. [8]

7. Consider equation (3).

$$\sigma = \frac{ne^2\tau}{m} \tag{3}$$

   i.) Derive equation (3) using Ohm's law as a starting point. [4]
   ii.) Derive equation (3) using the classical approach by assuming a free-electron gas under the influence of a viscous force. [4]

8. Copper has a density of 8.95 g/cm^3, atomic mass of 63 and a room-temperature electrical conductivity of $1.55 \times 10^{-8}\ \Omega \cdot m$. Calculate

   i.) the concentration of conduction electrons. [2]
   ii.) relaxation time, τ. [2]
   iii.) the Fermi level, E_F in electron-volts. [2]
   iv.) the Fermi velocity, v_F. [2]

   (Assume that the effective mass of the electron is the rest mass).

9. Discuss the failing(s) of the *free-electron* model. [8]

10. i.) In what essential way does a plasma differ from a gas? [2]
    ii.) Describe Matthiessen's law, the "Umklapp" process and the Lorentz number. [6]

Some general information and data

Planck's constant	$h = 6.626 \times 10^{-34}\ J \cdot s$
	$\hbar = h/2\pi$
Avogadro's constant	6.022×10^{23} mol^{-1}
Electronic charge	$e = 1.6022 \times 10^{-19}$ C
Electron mass	$m_e = 9.109 \times 10^{-31}$ kg
Neutron mass	$m_n = 1.675 \times 10^{-27}$ kg
Boltzmann's constant	$k_B = 1.38 \times 10^{-23}\ J/K$
Electron concentration	$N = Z_\nu \rho N_A / M$
Debye frequency	$\omega_D = \nu\left(6\pi^2 n\right)^{\frac{1}{3}}$
Debye temperature	$\theta_D = \hbar\omega_D / k_B$
Fermi energy	$\varepsilon = \frac{\hbar^2}{2m}\left(3\pi^2 n\right)^{\frac{2}{3}}$
A useful integral:	$\int_0^\infty \frac{x^3}{e^x - 1} dx = \frac{\pi^4}{15}$