FSK324 Test 2 - 2008

Time: 120 minutes	***Answer <u>ALL</u> questions.***	**Total marks: [90]**

Section A - The thermal properties of materials:

1. For a discrete lattice, the displacement of the r-th atom is

$$u_r = Ae^{i(kra-\omega t)}$$

where a is the lattice parameter, and the other symbols have their usual meanings. If the force on the r-th atom is

$$F_r = \mu(u_{r+1} + u_{r-1} - 2u_r),$$

determine

i.) the angular frequency ω in the form [8]

$$\omega^2 = \frac{4c}{M}\sin^2\varphi,$$

defining c and φ in terms of the known quantities.

ii.) Plot a graph of ω versus k [2]

iii.) Give the physical meaning of what happens if the wave vector ends on the boundary of the first Brillouin zone. [2]

iv.) Explain the meanings of the symbols c and k. [4]

2. Starting from the three-dimensional wave equation $u = Ae^{\bar{k}\cdot\bar{r}}$, show that the density of states for elastic vibrations in a three-dimensional structure is

$$g(\omega) = \frac{3V\omega^2}{2\pi^2\nu^3}$$

where the symbols have the normal meanings. [8]

3. The wave equation for elastic waves in a rod is given by

$$Yu_{xx} = \rho u_{tt}.$$

i.) Define *all* the symbols and their S.I. units in equation above. [2]

ii.) assuming a solution of the form $u = u(x,t) = Ae^{i(kx-\omega t)}$, show that the dispersion relation is $\omega = v_s k$ where $v_s = \sqrt{Y/\rho}$. [4]

iii.) Discuss the limits of the dispersion relation in a real, discrete lattice [2]

4. i.) Determine the change in thermal energy of 100g of copper (M=63.5, θ_D=348K) if it is cooled to 4K from (a) 300K, (b) 78K and (c) 20K [6]

   ii.) If the latent heat of liquid helium is 2700Jl^{-1}, calculate how much helium is needed to bring about the cooling in each case above. [6]

5. Determine the Debye temperature (θ_D) of gold given the following information: for gold, M=197, density is 1.9×10^4kg.m^{-3}, speed of sound is 2100m.s^{-1}. For copper, M=63.5, density is 8.9×10^3kg.m^{-3}, speed of sound is 3800m.s^{-1} and θ_D=348K. [6]

Section B - The free–electron model

6. Describe the Hall effect in metals. [8]

7. The electrical conductivity (σ) in a material is given by

$$\sigma = \frac{ne^2\tau}{m}$$

   i.) Derive the above equation using Classical theory (Ohm's law). [4]

   ii.) Derive the above equation by assuming a free-electron gas under the influence of a viscous force. [4]

8. Copper has a density of 8.95 g/cm^3, atomic mass of 63 and a room-temperature electrical conductivity of $1.55 \times 10^{-8}\ \Omega \cdot m$. Calculate

   i.) the concentration of conduction electrons. [2]

   ii.) relaxation time, τ. [2]

   iii.) the Fermi level, E_F in electron-volts. [2]

   iv.) the Fermi velocity, v_F. [2]

   (Assume that the effective mass of the electron is the rest mass).

9. i.) Plot the Fermi-Diràc distribution function $f(E)$ at room temperature (25^{o}C) and at 330K. [4]

   ii.) Assuming that the Fermi level ε_F=5eV is independent of temperature, calculate the energies E for which $f(E)$=0.5, 0.7, 0.9 and 0.95; [8]

10. i.) In what essential way does a plasma differ from a gas? [2]

    ii.) Distinguish between Matthiessen's law and the Kondo effect. [2]

Useful data

Planck's constant	$h = 6.626 \times 10^{-34}\ J \cdot s$
	$\hbar = h/2\pi$
Avogadro's constant	$6.022 \times 10^{23}\ \mathrm{mol}^{-1}$
Electronic charge	$e = 1.6022 \times 10^{-19}$ C
Electron mass	$m_e = 9.109 \times 10^{-31}$ kg
Neutron mass	$m_n = 1.675 \times 10^{-27}$ kg
Boltzmann's constant	$k_B = 1.38 \times 10^{-23}\ J/K$
Electron concentration	$N = Z_\nu \rho N_A / M$
Debye frequency	$\omega_D = \nu \left(6\pi^2 n\right)^{\frac{1}{3}}$
Debye temperature	$\theta_D = \hbar \omega_D / k_B$
Fermi energy	$\varepsilon = \frac{\hbar^2}{2m}\left(3\pi^2 n\right)^{\frac{2}{3}}$
A useful integral:	$\int_0^\infty \frac{x^3}{e^x - 1} dx = \frac{\pi^4}{15}$