

FSK324 Test 2

Solid State Physics

October 2009

Time: 90 minutes:	Answer ALL questions	Total marks: [60]
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Section A - The thermal properties of materials - 30 marks

1. For a periodic lattice having atomic masses M_1 and M_2 separated a distance a , the dispersion relation is given by

$$\omega^2 = \mu \left(\frac{1}{M_1} + \frac{1}{M_2} \right) \pm \mu \sqrt{\left(\frac{1}{M_1} + \frac{1}{M_2} \right)^2 - \frac{4 \sin^2 ka}{M_1 M_2}}.$$

- i.) Sketch and label a diagram that shows the optical and acoustic branches. [4]
- ii.) Discuss the onset of the optical and acoustic branches in terms of the amplitudes A_1 and A_2 of the wave solutions. [6]
2. With respect to specific heat capacity C_v , discuss the failings of
- i.) the classical Petit-Dulong model [3]
- ii.) the Einstein model [2]
3. Determine
- a.) the change in thermal energy of 100g of copper ($M=63.5$; $\theta_D=348\text{K}$) if it is cooled from 300K to 4k. [4]
- b.) the amount of helium (latent heat 2700 J/l) required to bring about the cooling in part (a.). [2]
4. Starting from the solution of the wave equation $u = u(x, t) = Ae^{i(kx - \omega t)}$,
- i.) show that the dispersion relation has the form

$$\omega = \pm \omega_m \sin \frac{ka}{2},$$

where a is the lattice parameter. [6]

- ii.) sketch the dispersion relation in the short-, medium- and long-wavelength regions. [3]

Section B - The free-electron model - 30 marks

5. Describe the Hall effect in metals using diagrams as necessary. Derive an expression for the Hall coefficient, R_H . [8]

6. The energy of a system in the Einstein model is

$$E = 3N_A \frac{\hbar\omega}{e^{\hbar\omega/kT} - 1}.$$

Determine

i.) the specific heat, C_v . [4]

i.) C_v at low temperatures [3]

ii.) C_v at high temperatures [3]

7. Starting from the Fermi energy (*see tables*), derive an expression for the density of states $D(\varepsilon)$ of electrons in terms of the concentration N per unit volume and energy ε . [8]

8. Explain the following terms

i.) Matthiesen's rule [4]

ii.) The Kondo effect [2]

iii.) An "Umklapp" process [4]

iv.) Lorentz number [2]

Some general information and data

Planck's constant	$h = 6.626 \times 10^{-34} \text{ J} \cdot \text{s}$ $\hbar = h/2\pi$
Avogadro's constant	$6.022 \times 10^{23} \text{ mol}^{-1}$
Electronic charge	$e = 1.6022 \times 10^{-19} \text{ C}$
Electron mass	$m_e = 9.109 \times 10^{-31} \text{ kg}$
Neutron mass	$m_n = 1.675 \times 10^{-27} \text{ kg}$
Boltzmann's constant	$k_B = 1.38 \times 10^{-23} \text{ J/K}$
Electron concentration	$N = Z\nu\rho N_A/M$
Debye frequency	$\omega_D = \nu(6\pi^2 n)^{\frac{1}{3}}$
Debye temperature	$\theta_D = \hbar\omega_D/k_B$
Fermi energy	$\varepsilon_F = \frac{\hbar^2}{2m} (3\pi^2 n)^{\frac{2}{3}}$
A useful integral:	$\int_0^\infty \frac{x^3}{e^x - 1} dx = \frac{\pi^4}{15}$