FSK324 Test 2
Solid State Physics

October 2011

Time: 2 hours:	Answer ALL questions	Total marks: [80]

Section A: Thermal properties

1. The average energy per atomic oscillator is given by

$$\bar{\varepsilon} = \sum_{n=0}^{\infty} \varepsilon_n e^{-(\varepsilon_n/kT)} \Big/ \sum_{n=0}^{\infty} e^{-(\varepsilon_n/kT)}$$

Given that the energy of an isolated oscillator is ε_n=$n\hbar\omega$ for n=0,1,2, ...

i.) Show that in closed form the energy can also be written as [4]

$$\bar{\varepsilon} = \frac{\hbar\omega}{e^{\hbar\omega/kT} - 1}$$

ii.) Hence derive an expression for the specific heat at constant volume (C_v) per mole. [6]

2. Starting from the solution of the wave equation in one dimension,

i.) Apply the Born-von Karman (BVK) boundary value condition and derive an expression for allowable values of the wave vector k [6]

ii.) hence derive an expression for the number of vibrational modes for frequencies between ω and ω+$d\omega$ [2].

iii.) What is the physical significance of the BVK condition? [2]

3. Silicon has a molar mass of 28.09 and Debye temperature of 647K. Determine

i.) the change in thermal energy of 400g of silicon if it is cooled from 300K to 10k. [6]

ii.) the amount of helium (latent heat 2700 J/l) required to bring about the cooling above. [2]

iii.) Calculate the angular frequency of the phonon vibration in silicon. [2]

4. Starting from the solution of the wave equation $u = u(x, t) = Ae^{i(kx-\omega t)}$,

i.) show that the dispersion relation has the form

$$\omega = \pm\omega_m \sin\frac{ka}{2},$$

where a is the lattice parameter. [6]

ii.) sketch the dispersion relation in the short-, medium- and long-wavelength regions. [3]

5. For a periodic lattice having atomic masses M_1 and M_2 separated a distance a, the dispersion relation is given by

$$\omega^2 = \mu\left(\frac{1}{M_1} + \frac{1}{M_2}\right) \pm \mu\sqrt{\left(\frac{1}{M_1} + \frac{1}{M_2}\right)^2 - \frac{4\sin^2\ ka}{M_1 M_2}}.$$

   i.) State the conditions that give the optical branch *and* the acoustic branch in the above equation. [2]

   ii.) Sketch an accurate, labeled graph showing both the acoustic and optical branches with all the limiting values. [4]

   iii.) Discuss the dynamic difference between the acoustic branch and the optical branches by comparing their behavior at k=0. *Hint*: use the relative vibrational amplitudes of the non-homogeneous masses M_1 and M_2. [4]

6. i.) Determine the Debye temperature of FCC gold whose atomic mass is 197, density is 1.93×10^4 kg/m^3. Gold propagates sound at 2100 m/s. On the other hand FCC copper has atomic mass 63.5, Debye temperature of 348K, density 8900 kg/m^3 and propagates sound at 3800 m/s. [4]

   ii.) Discuss the thermal conductivity in non-electrical conductors by describing only the contribution of phonon interactions in three temperature regions: T=0K, low-temperatures, and high-temperatures. *Hint*: consider the interactions of $\bar{k}_1$ and $\bar{k}_2$ and their resultant in the Brilloiun zones. [6]

Section B: Free-electron model

7. The current density J flowing in an electrical conductor of length l, cross-sectional area A under an electrical potential V (or electric field ξ) is given by J=$\sigma\xi$, where σ is the electrical conductivity.

   i.) Show that J=$\sigma\xi$ is indeed Ohm's law, I=V/R. [2]

   ii.) Show that the electrical conductivity is given by

$$\sigma = \frac{Ne^2\tau}{m},$$

   where n is electron density (number per unit volume), τ is the relaxation time and m is electron mass. [4]

   iii.) Copper (valency 2) has density 8.95 g/cm^3, electrical resistivity 1.55 $\times10^{-8}\Omega$m at room temperature. Determine electron relaxation time in copper (II). [4]

8. Explain the following terms

   i.) Matthiesen's rule [4]

   ii.) The Kondo effect [4]

   iii.) Lorentz number [2]

Useful information

Planck's constant	$h = 6.626 \times 10^{-34}\ Js$
	$\hbar = h/2\pi$
Avogadro's constant	6.022×10^{23} mol^{-1}
Electronic charge	$e = 1.6022 \times 10^{-19}$ C
Electron mass	$m_e = 9.109 \times 10^{-31}$ kg
Neutron mass	$m_n = 1.675 \times 10^{-27}$ kg
Boltzmann's constant	$k_B = 1.38 \times 10^{-23}\ J/K$
Electron concentration	$N = Z_\nu \rho N_A/M$
Debye frequency	$\omega_D = \nu\left(6\pi^2 n\right)^{\frac{1}{3}}$
Debye temperature	$\theta_D = \hbar\omega_D/k_B$
Fermi energy	$\varepsilon_F = \frac{\hbar^2}{2m}\left(3\pi^2 n\right)^{\frac{2}{3}}$
	$\int_0^\infty \frac{x^3}{e^x-1}dx = \frac{\pi^4}{15}$