

FSK324 Test 2

Solid State Physics

20th October 2012

Time: 2 hours:

Answer ALL questions

Total marks: [80]

Section A: Thermal properties

1. The average energy per atomic oscillator is given by

$$\bar{\varepsilon} = \frac{\sum_{n=0}^{\infty} \varepsilon_n e^{-(\varepsilon_n/kT)}}{\sum_{n=0}^{\infty} e^{-(\varepsilon_n/kT)}}$$

Given that the energy of an isolated oscillator is $\varepsilon_n = n\hbar\omega$ for $n=0,1,2, \dots$

- i.) Show that [4]

$$\bar{\varepsilon} = \frac{\hbar\omega}{e^{\hbar\omega/kT} - 1}$$

- ii.) Hence derive an expression for C_v . [6]

2. The Debye temperature of NaCl and KCl, which have the same crystal structure, are 310K and 230K respectively. The specific heat of KCl at 5K is 3.8×10^{-2} J/K/mol. Calculate the specific heat of NaCl at 5K and that of KCl at 2K. [8]

3. Starting from the solution of the wave equation $u = u(x, t) = Ae^{i(kx - \omega t)}$,

- i.) show that the dispersion relation has the form

$$\omega = \pm \omega_m \sin \frac{ka}{2},$$

where a is the lattice parameter. [6]

- ii.) sketch and label the dispersion relation in the short and long wavelength regions. [2]

4. Determine the Debye temperature of FCC gold whose atomic mass is 197, density is 1.93×10^4 kg/m³. Gold propagates sound at 2100 m/s. On the other hand FCC copper has atomic mass 63.5, Debye temperature of 348K, density 8900 kg/m³ and propagates sound at 3800 m/s. [6]

Section B: Free-electron model

5. Suppose that a conductor under electrical potential V experiences current density J . If the conductor has length l and cross-sectional area A

- i.) Derive an expression of Ohm's law in terms of J , conductivity σ and field ξ . [4]

ii.) Show that

$$\sigma = \frac{ne^2\tau}{m},$$

where n is the number of electrons per unit volume, τ is electron relaxation time and m is electron rest mass. [4]

iii.) The following questions apply to copper, valency of one, molar mass is 63.5g, density is 8.95 g/cm³ and electrical resistivity is 1.55×10^{-8} (Ωm). Determine:

a) the concentration of conduction electrons [4]

b) the electron relaxation time. [4]

c) the fermi energy [4]

d) the mean free path at the fermi level. [4]

6. Considering both the classical Drude theory and the quantum mechanical free electron Fermi gas theory with regard to electrical conductivity,

i.) List three successes and three failures of the classical theory. [6]

ii.) List three failings of the free electron gas theory. [3]

iii.) The following questions apply to the fermi level of sodium, whose fermi energy is 3.23 eV.

a) Explain what is meant by the fermi energy [2]

b) Calculate the fermi temperature of sodium. [4]

c) Use tabulated data to plot the Fermi-Dirac (FD) distribution for sodium at 300 K. [6]

d) Determine the energy for which the FD probabilities are: 0.5, 0.8 and 0.98. [3]

Useful information

Planck's constant	$h = 6.626 \times 10^{-34} \text{ Js}$
	$\hbar = h/2\pi$
Avogadro's constant	$6.022 \times 10^{23} \text{ mol}^{-1}$
Electronic charge	$e = 1.6022 \times 10^{-19} \text{ C}$
Electron mass	$m_e = 9.109 \times 10^{-31} \text{ kg}$
Neutron mass	$m_n = 1.675 \times 10^{-27} \text{ kg}$
Boltzmann's constant	$k_B = 1.38 \times 10^{-23} \text{ J/K}$
Electron concentration	$n = Z\nu\rho N_A/M$
Debye frequency	$\omega_D = \nu(6\pi^2n)^{\frac{1}{3}}$
Debye temperature	$\theta_D = \hbar\omega_D/k_B$
Fermi energy	$\varepsilon_F = \frac{\hbar^2}{2m}(3\pi^2n)^{\frac{2}{3}}$
	$\int_0^\infty \frac{x^3}{e^x-1} dx = \frac{\pi^4}{15}$